R18

Code No: 153CH

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech II Year I Semester Examinations, February - 2024
MATHEMATICAL AND STATISTICAL FOUNDATIONS
(Common to CSE(CS), CSE(AI&ML), CSE(DS), AI&ML)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) State the Chinese reminder theorem. [2]

b) Find the number of distinct solutions of $15x = 25 (\text{mod } 35)$. [3]

c) If $x = 4y + 5$, $y = kx + 4$ are the regression lines of x on y and y on x respectively, then show that $k \in [0, 0.5]$. [2]

d) The probability density function of a continuous random variable X is given by $P(x) = \alpha\, e^{-|x|}$, where $-\infty < x < \infty$. Prove or disprove that $\alpha = 0.5$. [3]

e) If z is normally distributed with mean 0 and variance 1, obtain P ($-1.5 \le z \le 1.5$). [2]

f) Explain the sample distribution with supporting example. [3]

g) If $n = 100$, $\sigma = 5$, compute the maximum error with 95% confidence limits. [2]

h) What is meant by interval estimation? [3]

i) Define Markov chain. [2]

j) Is the matrix $\begin{pmatrix} 1/3 & 2/3 & 4/3 \\ 1/2 & 1 & 1/2 \end{pmatrix}$ stochastic? [3]

PART – B

(50 Marks)

2.a) If $d = \gcd(826, 1890)$, find d using Euclidean algorithm and express d as a linear combination of 826 and 1890.

b) Show that $\sqrt{2}$ is not a rational number. [6+4]

OR

3.a) If p is a prime and p does not divide a, then prove that $a^{p-1} \equiv 1 (\text{mod } p)$, where a is an integer.

b) Solve the simultaneous congruence's: $x \equiv 2 (\text{mod } 7)$ and $x \equiv 7 (\text{mod } 16)$. [5+5]

4. Estimate the coefficient of correlation from the following data:

x	10	12	13	16	17	20	25
y	10	22	24	27	29	33	37

Also obtain two lines of regression. [10]

OR

5.a) Given that $P(X=2)=9P(X=4)+90P(X=46)$ for a Poisson variate X, calculate the probability that X < 2.

b) A random variable X has the following probability function:

x:	0	1	3	4	5	6	7
$f(x)$:	0	k	2k	2k	3k	k^2	$7k^2+k$

Determine the value of k, (ii) P(0 < X < 5). [4+6]

6.a) Suppose that on the average one person in 1000 makes a numerical error in preparing income tax return. If 10,000 forms are selected at random and examined, find the probability that 6, 7 or 8 of the forms will be in error.

b) When the mean of marks was 50% and standard deviation 5% then 60% of the students failed in an examination. Estimate the 'grace' marks to be awarded in order to show that 70% of the students passed. Assume that the marks are normally distributed. [5+5]

OR

7. A population consists of the five numbers 2, 3, 6, 8, and 11. Consider all possible samples of size 2 that can be drawn without replacement from this population. Find (a) the mean of the population, (b) the standard deviation of the population, (c) the mean of the sampling distribution of means, and (d) the standard deviation of the sampling distribution of means. [2+3+2+3]

8. The pulse rate of 50 yoga practitioners decreased on the average by 20.2 beats/min. with standard deviation of 3.5.

a) If $\bar{x}=20.2$ is used as a point estimate of the true average decrease in the pulse rate, what can we assert with 95% confidence about the maximum error E?

b) Construct 99% confidence interval for the true average decrease in pulse rate. [5+5]

OR

9. The length of life X of certain computers is approximately normally distributed with mean 800 hours and standard deviation 40 hours. If a random sample of 30 computers has an average life of 788 hours, test the null hypothesis that μ = 800 hours against the alternative that $\mu \neq 800$ hours at (a) 0.5%, (b) 5% level of significance. [5+5]

10. a) Compute the unique fixed probability vector t of $P=\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$.

b) What matrix does P^n approach?

c) What vector does $\begin{pmatrix} 0.25 & 0.75 \end{pmatrix} P^n$ approach? [4+3+3]

OR

11. Compute the steady state vector for the transition matrix $P=\begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.1 & 0.4 & 0.5 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$. [10]

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